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PROPERTIES OF DENSITY TAPERED ARRAYS

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TITLE: PROPERTIES OF DENSITY TAPERED ARRAYS

AUTHOR: G J Ball

DATE: March 1988

SUMMARY

The properties of density tapered arrays designed with deterministic and statistical techniques are reviewed and compared. The properties of statistically designed arrays are then described as a function of main beam scan, average sidelobe level, model amplitude distribution, thinning, and array and aperture size. Firstly it was found that without additional signal processing large apertures were needed to produce low sidelobe levels. For example, the minimum circular aperture needed to produce a −40 dB peak sidelobe level has a diameter of about 178 wavelengths. Secondly it emerged that statistical density tapering should be used to design arrays for applications with scanned main beams, since their peak sidelobe levels remain approximately constant as the main beam is scanned. The sidelobe levels of the deterministic array increase as the mainbeam is scanned.

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PROPERTIES OF DENSITY TAPERED ARRAYS

G J Ball

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1 INTRODUCTION

The two most commonly used techniques for designing density tapered arrays[1] – deterministic[2] and statistical[3] – are reviewed and compared. The properties of statistical density tapered arrays are then described as a function of main beam scan, average sidelobe level, model amplitude distribution, and array and aperture size.

A density tapered array is a thinned array of uniformly weighted elements placed in an aperture such that their density changes to approximate a model amplitude distribution. This model amplitude distribution is one known to produce low sidelobes if applied to a fully filled array (ie one with elements placed at the intersections of a square grid with a half wavelength spacing in both directions).

Density tapering offers the advantages of reductions in both cost and complexity, because the arrays are thinned (ie they contain fewer elements than a fully filled array) and all the elements have the same amplitude weight. It also gives rise to the disadvantages of a reduction in antenna gain and an increase in the proportion of the total power radiated or received in the sidelobes. Thinned, density tapered arrays have been built that achieve −35 dB peak sidelobes with only 50% of the elements of a fully filled array[4]. Arrays with considerable thinning (90% of elements removed) have also been constructed.

To design deterministic arrays the total volume under the model amplitude distribution is divided into smaller, equal volumes, with one per element of the thinned array. The element is then placed somewhere within the projection of this volume on to the aperture plane. In the Willey method[2], for example, a circular aperture is divided

into a number of concentric rings of a half wavelength radial width. Elements are placed halfway between the inner and outer edges of these rings, with equal angular intervals between them. To density taper the array the number of elements in each ring is given by the following:

$$\frac{\text{Number of elements per ring}}{\text{Total number of elements}} = \frac{\text{Volume of model amplitude distribution integrated over ring}}{\text{Total volume integrated over aperture}}$$

An example element distribution is shown in Figure 1a.

Statistical density tapering[3] uses the model amplitude distribution as a probability density function for specifying the locations of the elements. An array is constructed by first setting up a grid of possible element locations, defined by the intersection points of a square grid with a half wavelength spacing in both directions. For each location a random number between zero and unity is generated and compared with the product qA. Here A is the value of the model amplitude distribution at the possible location, and q is a factor between zero and unity that determines the degree of thinning. An array with q equal to unity is termed naturally thinned. Greater thinning is achieved by setting q to a value of less than unity. If the random number is less than qA an element is placed at the location, otherwise it is left unoccupied. The technnique is equivalent to removing elements from a fully filled array to leave the required variation in element density, see Figure 2a. As this is a statistical method it requires enough samples to ensure statistical regularity. Empirically it has been found that arrays of greater than 100 elements are adequate for this technique[1].

2 ARRAY PROPERTIES

2.1 RADIATION PATTERNS

The radiation patterns of an equivalent deterministic and naturally thinned statistical array were computed and are compared in Figures 1b and 2b. The pattern of an equal-sized aperture fully filled, fully amplitude tapered array (ie an array of elements each with an amplitude exactly equal to A) is also shown in Figures 1b and 2b for comparison. The radiation patterns are for arrays of isotropic point sources centred on the origin in the XY plane. A normalised 40 dB (n=4) circular Taylor distribution was used both as the model amplitude distribution for the density tapered arrays, and to give the weights for the fully filled array. All the arrays were designed to radiate or receive the same power in the main beam direction, and so are normalised to the same value in Figures 1b and 2b.

All of the patterns are similar in the main beam region, as this is primarily determined only by aperture size[1]. The sidelobes of the density tapered arrays are much higher than those of the fully filled, fully amplitude tapered array. The sidelobes of the deterministic array are oscillatory, with the peak values increasing away from the main beam. The statistical array has randomly varying sidelobes.

The radiation pattern of a statistical array is the sum of two terms[3]: one proportional by a factor of q^2 to the pattern of a fully filled, fully amplitude tapered array; the other the average level of the random sidelobes. The first term is much larger in the main beam region, so that the main beams of statistical and fully filled, fully amplitude tapered arrays are similar. The second term is larger in the sidelobe region, so that the sidelobes of statistical arrays are higher than those of fully filled arrays.

2.2 PROPERTIES AS A FUNCTION OF MAIN BEAM SCAN

The peak sidelobe level of a statistically density tapered array remains approximately constant as its main beam is scanned, while that of a deterministic array increases. Hence statistically designed arrays are better for scanning application. This is demonstrated in Figure 3, which compares the radiation patterns of statistical and deterministic arrays with their main beams at 0° and −60°. The peak sidelobe level of the deterministic array rose from −26 dB to −16.7 dB as the main beam was scanned from 0° to −60°, while that of the statistical array did not significantly alter, changing from −21.92 dB to −21.95 dB. Only statistical arrays are considered in the rest of this Memorandum.

2.3 COMPARISON OF METHODS FOR PREDICTING THE AVERAGE SIDELOBE LEVEL

Four equations for predicting the average sidelobe level S have been reported in the literature:

EQUATION	REFERENCE	
$S = \dfrac{\sum_{i=1}^{M} qA_i(1-qA_i)}{\lvert E_o(\theta,\varphi)\rvert^2}$	Ref 2	(1)
$S = (1-K)/N$	Ref 5	(2)
$S = (1-K')/N$	Ref 6	(3)
$S = 1/N$	Refs 1 and 5	(4)

where:

A_i	is the value of A at possible element location i
$\lvert E_o(\theta,\varphi)\rvert^2$	is the peak radiated power of an equal aperture sized fully filled, fully amplitude tapered array
M	is the total number of elements if the aperture were fully filled
N	is actual number of elements in the array
K	is the fill factor, equal to N/M
K'	is a refined version of K, defined in Reference 6
q	is the thinning factor $0.0 < q < 1.0$.

To determine their accuracy values predicted from equations (1) to (4) were compared with directly calculated values from arrays of point sources. It was found that equations (1) to (3) accurately predicted the average sidelobe level. Equation (4), while less accurate, gives a quickly calculable estimate. The results are given in Table 1 as a function of aperture size, represented by M. (M is proportional to aperture area as each elements occupies an area of $0.25\ \lambda^2$, so that the total area is $0.25\ M\lambda^2$.)

Total number of elements if aperture fully filled, M	208	812	3228	7825
Number of elements in density tapered array, N	86	295	1251	3060
Average sidelobe level directly calculated from radiation patterns	-21.3	-26.9	-34.6	-38.6
Average sidelobe level predicted from eqn (1)	-22.5	-28.6	-34.6	-38.4
Average sidelobe level predicted from eqn (2)	-21.7	-26.7	-33.1	-37.1
Average sidelobe level predicted from eqn (3)	-22.7	-28.6	-34.6	-38.5
Average sidelobe level predicted from eqn (4)	-19.4	-24.7	-31.0	-34.9

TABLE 1. Comparison of Methods for Predicting the Average Sidelobe Level

Model Amplitude Distribution: 40 dB (n=4) Taylor

2.4 PROPERTIES AS A FUNCTION OF THE MODEL AMPLITUDE DISTRIBUTION

Three different model amplitude distributions, all with parameters set to give a −40 dB sidelobe level if applied to a fully filled array, were used to design statistical density tapered arrays. It was found that all three produced very similar main beamwidths and peak sidelobe levels, indicating that the choice of amplitude distribution is not significant (provided it is chosen to give the required sidelobe level). The three distributions were a 40 dB (n=4) Taylor distribution, a 40 dB Hansen distribution, and a $(1-r/R)^n + b$ distribution.

2.5 ARRAY PROPERTIES AS A FUNCTION OF THINNING

As an array is thinned the number of elements is reduced and the sidelobe level is increased. This is illustrated in Figure 4.

2.6 PROPERTIES AS A FUNCTION OF APERTURE SIZE

The average and peak sidelobe levels of an array are reduced as the aperture size is increased. This is because the number of elements in the array is increased so that the change in element density gives a better approximation to the model amplitude distribution. This is shown in Figure 5, which plots the average sidelobe levels of a naturally thinned array calculated from equation (3) as a function of aperture size. The aperture size is represented by M. Results are given for Taylor distributions with design sidelobe levels of −25, −30, −35 and −40 dB. The peak sidelobe levels are also plotted, and were determined by adding 10 dB to the

average sidelobe level. This is a convenient and reasonably accurate rule-of-thumb[7].

Figure 5 also shows that a minimum aperture size is necessary to achieve a required sidelobe level (without additional processing). And that this minimum size is increased as the required sidelobe level is reduced, so that large apertures (in terms of wavelength) are needed to produce −40 dB sidelobe levels. Table 2 gives the minimum naturally thinned array aperture size and the minimum number of elements needed to achieve sidelobe levels between −25 and −40 dB. For example, the minimum circular aperture needed to produce a −40 dB peak sidelobe level has a diameter of about 178 wavelengths, and contains 39,000 elements.

Required Peak Sidelobe Level/ dB	Approximate Number of Elements in Aperture if Fully Filled M	Circular Aperture Diameter/ Wavelengths	Actual Number of Elements in Array, N
-25	1,930	25	1,150
-30	8,100	51	4,100
-35	30,760	99	13,600
-40	100,000	178	39,000

TABLE 2. Minimum Aperture Sizes

To validate these results Figure 6 shows the radiation pattern of a naturally thinned array with the minimum aperture size and number of elements necessary to produce a −30 dB peak sidelobe level. Apart from the first sidelobe (near the main beam where the term proportional to the pattern of a fully filled array contributes) all the sidelobes are below −30 dB. These results are for naturally thinned arrays. Arrays with greater thinning need larger minimum apertures to achieve required sidelobe levels.

3 CONCLUSIONS

There are two main conclusions. Firstly that large apertures are required to produce low sidelobe levels (without additional signal processing), and secondly that statistical density tapering should be used to design arrays for applications that require scanned main beams.

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4. E Brookner, "A Review of Array Radars", Microwave Journal, Vol 14, No 10, pp 25-44.

5. Halford, G J and McCullogh, W J, IEE Conference Publications No 195: Antennas and Propagation (Proc 2nd International Conf on Antennas and Propagation), Part 1: Antennas, pp 20-24.

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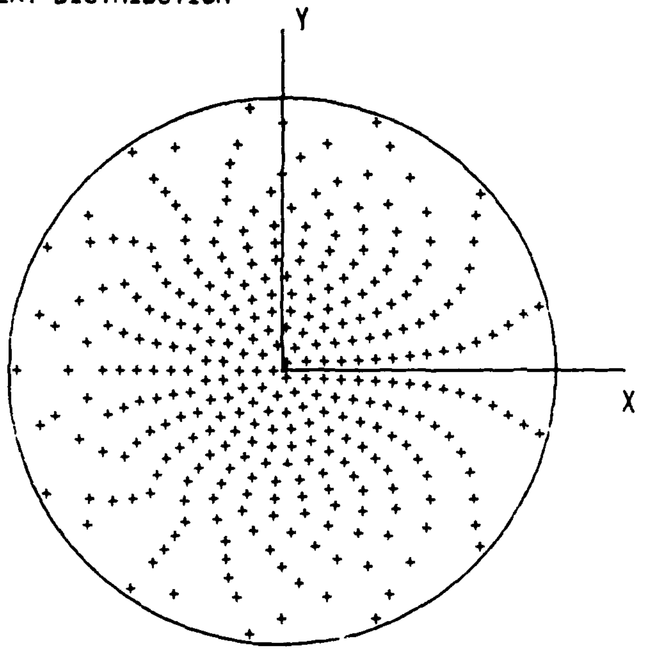
7. B D Steinberg, "Principles of Aperture and Array System Design".

FIG 1. DETERMINISTIC DENSITY TAPERING: WILLEY METHOD

1(a) ELEMENT DISTRIBUTION



1(b) REL POWER PATTERNS OF EQVLT AMP AND DENSITY TAPERED ARRAYS

Amplitude distribution: -40 dB (n=4) circular Taylor distribution
Aperture size: Circle of 16 wavelengths in diameter
Number of elements in density tapered array: 321
Number of elements in amplitude tapered array (fully filled): 804

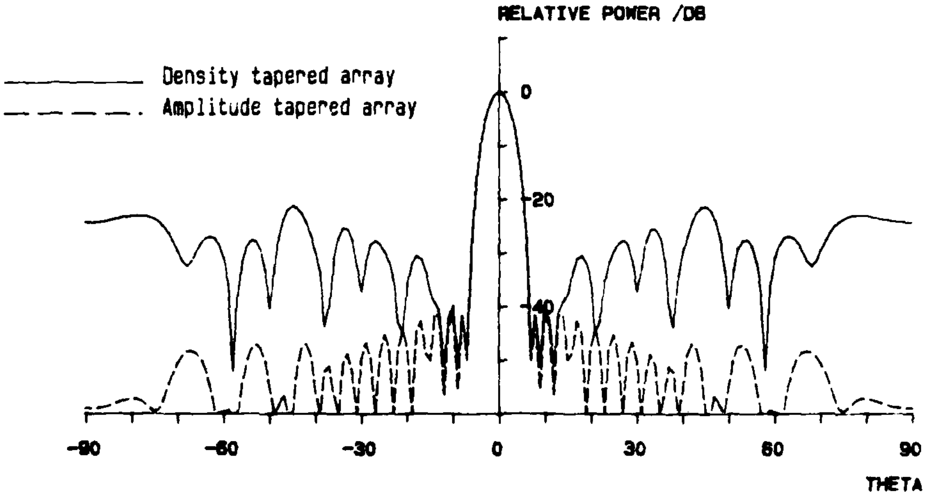
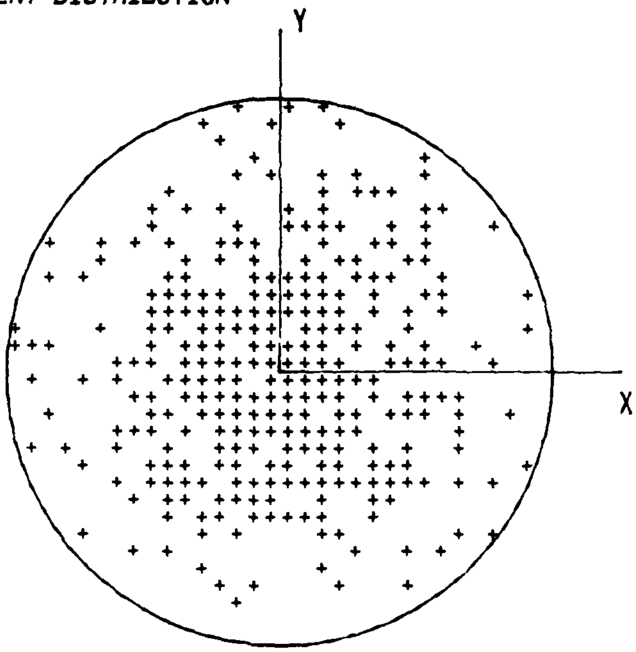


FIG 2. STATISTICAL DENSITY TAPERING

2 (a) ELEMENT DISTRIBUTION



2 (b) REL POWER PATTERNS OF EQVLT AMP AND DENSITY TAPERED ARRAYS

Amplitude distribution: -40 dB (n=4) circular Taylor distribution
Aperture size: Circle of 16 wavelengths in diameter
Number of elements in density tapered array: 295
Number of elements in amplitude tapered array (fully filled): 812

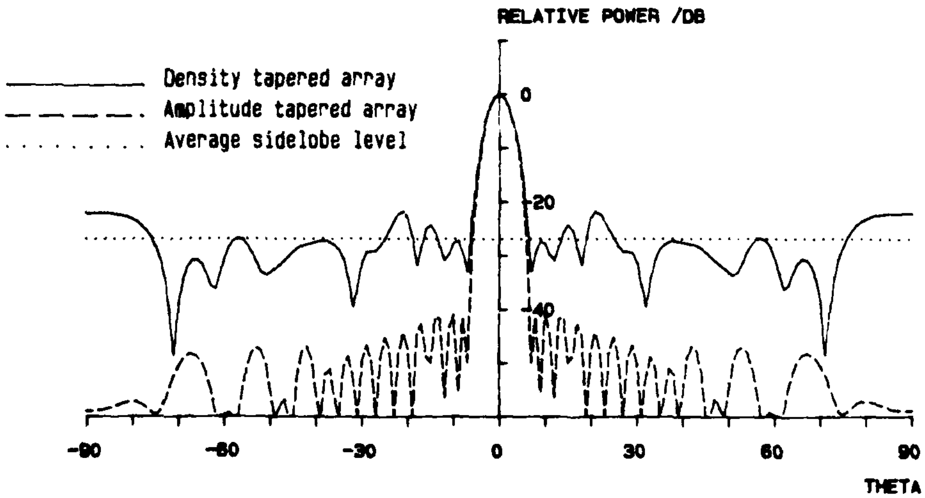


FIG 3. COMPARISON OF DETERMINISTIC AND STATISTICAL DENSITY TAPERING METHODS FOR ARRAYS WITH SCANNED MAIN BEAMS

Amplitude taper used: -40 dB circular Taylor distribution
Aperture size: Circle of 16 wavelengths in diameter

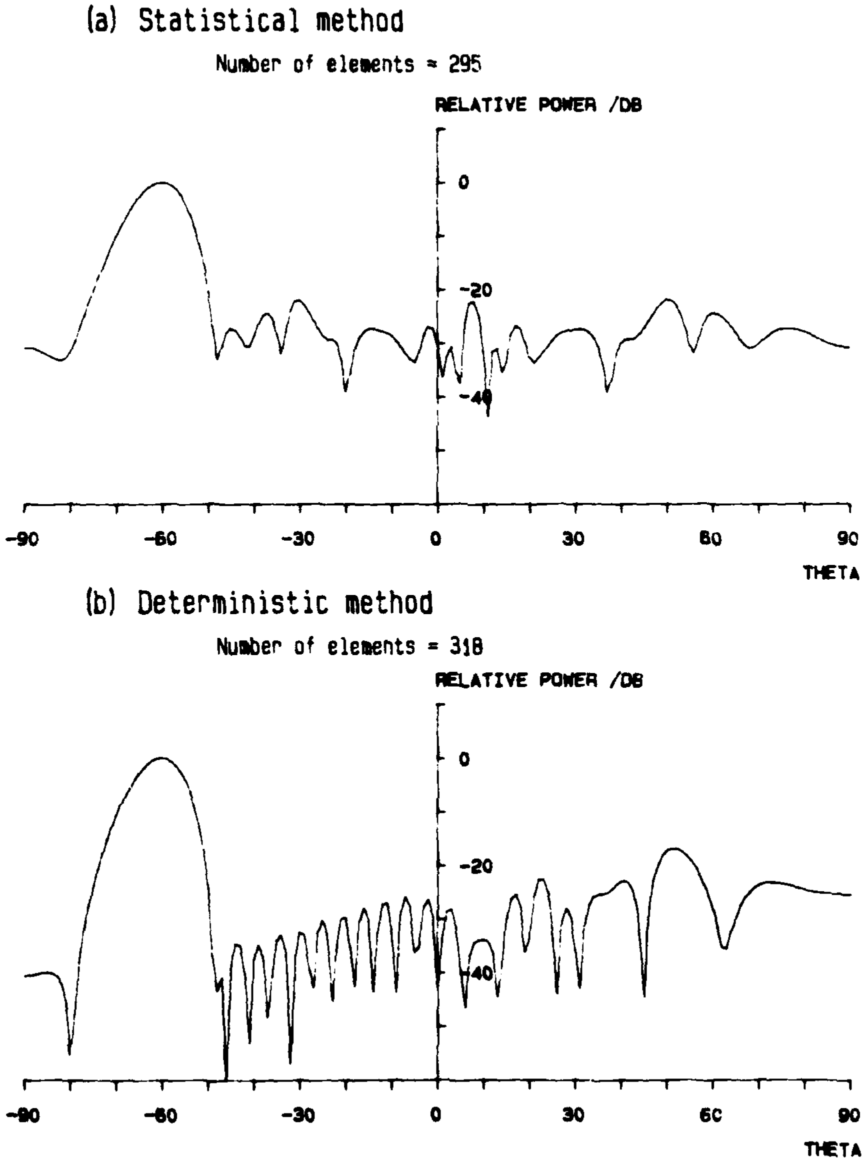


FIG 4. EFFECT OF A REDUCTION IN THE NUMBER OF ELEMENTS

Amplitude distribution: -30 dB (n=3) circular Taylor distribution
Aperture size: Circle of 16 wavelengths in diameter
Number of elements in array (q=1.0): 408
Number of elements in array (q=0.5): 203

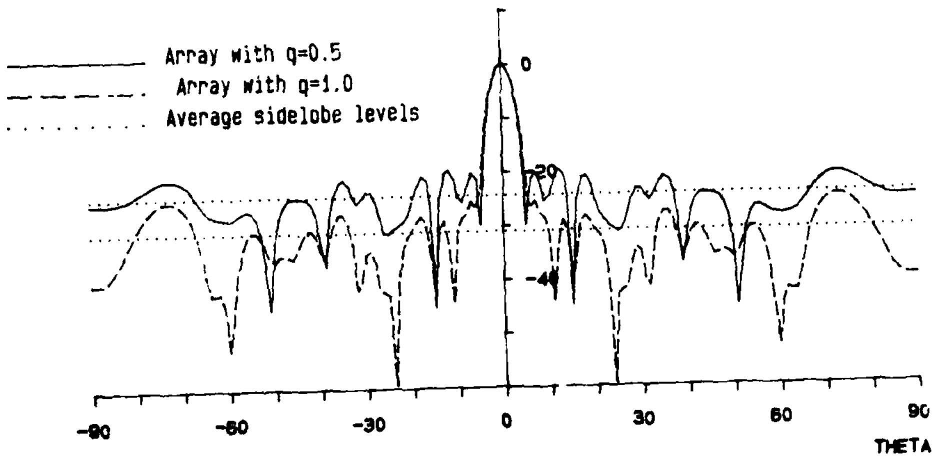


FIG.5. AVERAGE AND PEAK SIDELOBE LEVEL OF A NATURALLY THINNED ARRAY AS A FUNCTION OF APERTURE SIZE

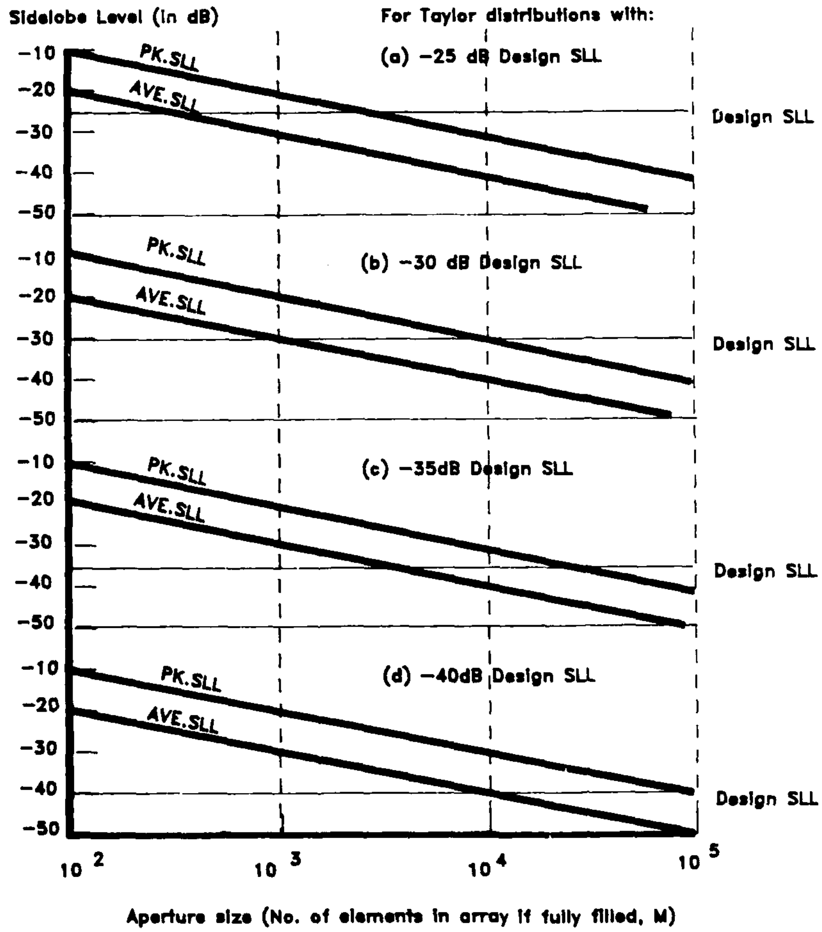
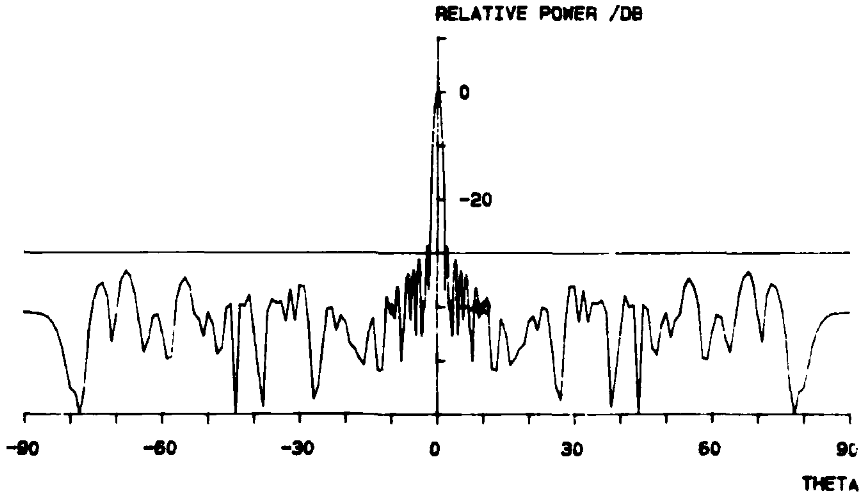


FIG 6. REL POWER PATTERN OF LARGE DENSITY TAPERED ARRAY

Amplitude distribution: -30 dB (n=3) circular Taylor distribution
Aperture size: Circle of 52 wavelengths in diameter
Number of elements in density tapered array: 4098
Number of elements in fully filled array: 8492



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Abstract

The properties of density tapered arrays designed with deterministic and statistical techniques are reviewed and compared. The properties of statistically designed arrays are then described as a function of mainbeam scan, average sidelobe level, model amplitude distribution, thinning, and array and aperture size. Firstly, it was found that without additional signal processing large apertures were needed to produce low sidelobe levels. For example, the minimum circular aperture needed to produce a -40 dB peak sidelobe level has a diameter of about 178 wavelengths. Secondly, it emerged that statistical density tapering should be used to design arrays for applications with scanned mainbeams, since their peak sidelobe levels remain approximately constant as the mainbeam is scanned. The sidelobe levels of the deterministic array increase as the mainbeam is scanned.

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